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Development and Validation of a General Purpose Linearization Program for Rigid Aircraft Models

Eugene L. Duke* and Robert F. Antoniewicz†

NASA Ames Research Center
Dryden Flight Research Facility
Edwards, California

Abstract

This paper discusses a FORTRAN program that provides the user with a powerful and flexible tool for the linearization of aircraft models. The program LINEAR numerically determines a linear systems model using nonlinear equations of motion and a user-supplied, nonlinear aerodynamic model. The system model determined by LINEAR consists of matrices for both the state and observation equations. The program has been designed to allow easy selection and definition of the state, control, and observation variables to be used in a particular model. Also, included in the report is a comparison of linear and nonlinear models for a high-performance aircraft.

Symbols

The units associated with the quantities listed below are expressed in a generalized system. Where applicable, quantities are defined with respect to the body-axis system.

Variables

A	state matrix of the equation, $\dot{x} = Ax + Bu$
A	axial force, lb
A'	state matrix of the equation, $C\dot{x} = A'x + B'u$
$a_{n,i}$	normal acceleration not at vehicle center of gravity, g
a_{n_x}	x-body axis accelerometer at vehicle center of gravity, g
$a_{n_{x,i}}$	x-body axis accelerometer not at vehicle center of gravity, g
a_{n_y}	y-body axis accelerometer at vehicle center of gravity, g
$a_{n_{y,i}}$	y-body axis accelerometer not at vehicle center of gravity, g
a_{n_z}	z-body axis accelerometer at vehicle center of gravity, g
$a_{n_{z,i}}$	z-body axis accelerometer not at vehicle center of gravity, g
a_x	acceleration along the x-body axis, g
a_y	acceleration along the y-body axis, g
a_z	acceleration along the z-body axis, g
B	control matrix of the equation, $\dot{x} = Ax + Bu$
B'	control matrix of the equation, $C\dot{x} = A'x + B'u$
C	C-matrix of the state equation $C\dot{x} = A'x + B'u$
D	dynamic interaction matrix for state equation, $x = Ax + Bu + Dv$
D'	dynamic interaction matrix for the state equation, $C\dot{x} = A'x + B'u + D'v$
D	drag force, lb
E	dynamic interaction matrix for the observation equation, $y = Hx + Fu + Ev$
E'	dynamic interaction matrix for the observation equation, $y = H'x + G\dot{x} + F'u + E'v$
E_s	specific energy, ft
F	feed-forward matrix of the equation, $y = Hx + Fu$
F'	feed-forward matrix of the observation equation, $\dot{y} = H'x + G\dot{x} + F'u$
f	nonlinear state vector function
fpa	flightpath acceleration, g
G	G-matrix of the observation equation, $y = H'x + Gx + F'u$
g	nonlinear observation vector function
H	observation matrix of the equation, $y = Hx + Fu$
H'	observation matrix of the equation, $y = H'x + G\dot{x} + F'u$

*Aerospace Engineer. Member AIAA.
†Aerospace Engineer.

h	altitude, ft
h,i	altitude not at vehicle center of gravity, ft
L	lift, lb
M	Mach number
N	normal force, lb
n	load factor
P_s	specific power, ft/sec
p	roll rate, deg
p_a	ambient pressure, lb/ft^2
p_s	stability axis roll rate, deg/sec
p_t	total pressure, lb/ft^2
q	pitch rate, deg/sec
$\bar{q}$	dynamic pressure, lb/ft^2
q_c	impact pressure, lb/ft^2
q_s	stability axis pitch rate, deg/sec
Re	Reynolds number
Re$'$	Reynolds number per unit length, ft^{-1}
r	yaw rate, deg/sec
r_s	stability axis yaw rate, deg/sec
T	ambient temperature, °R
T	total angular momentum, slug-ft^2/sec^2
T_t	total temperature, °R
t	time, sec
u	velocity in x-axis direction, ft/sec
u	control vector
u_o	control vector at analysis point
V	total velocity, ft/sec
v	dynamic interaction vector
v	velocity in y-axis direction, ft/sec
w	velocity in z-axis direction, ft/sec
X	total force along the x body axis, lb
x	state vector
x_o	state vector along nominal trajectory
Y	side force, lb
y	observation vector
y_o	observation vector along nominal trajectory
Z	total force along the z-body axis, lb
α	angle of attack, deg
α,i	angle of attack measurement not at vehicle center of gravity, deg
β	angle of sideslip, radians
β,i	angle of sideslip measurement not at vehicle center of gravity, deg
γ	flight path angle, deg
Δ	small perturbation value
δL	incremental rolling moment, ft lb
δM	incremental pitching moment, ft lb
δN	incremental yawing moment, ft lb
δX	incremental x-body axis force, lb
δY	incremental y-body axis force, lb
δZ	incremental z-body axis force, lb
θ	pitch angle, deg
ϕ	roll angle, deg
ψ	heading angle, deg

Superscripts

$\cdot$	indicates derivative with respect to time
T	transpose of a vector or matrix

Introduction

The program LINEAR, described in this paper, was developed at the Dryden Flight Research Facility of the NASA Ames Research Center to provide a standard, documented, and verified tool to derive linear models for aircraft stability analysis and control law design. This development was undertaken to eliminate the need for the aircraft-specific linearization programs common in the aerospace industry. The lack of available, documented linearization program provided a strong motivation for this paper and a NASA technical paper (to be published) on the development of LINEAR. Reference 1 represents the only available documented linearization program before LINEAR.

Linear system models of aircraft dynamics and sensors are an essential part of both vehicle-stability analysis and control law design. These models define the aircraft system in the neighborhood of an analysis point and are determined by the linearization of the nonlinear equations defining vehicle dynamics and sensors. This report describes a FORTRAN program that provides the user with a powerful and flexible tool for the linearization of aircraft models. LINEAR is a program with well-defined and generalized interfaces to aerodynamic and engine models and is designed to address a wide range of problems without the requirement of program modification.

The program LINEAR numerically determines a linear systems model using nonlinear equations of motion and a user-supplied nonlinear aerodynamic model. LINEAR is also capable of extracting both linearized engine effects (such as net thrust, torque, and gyroscopic effects) and including these effects in the linear system model. The point at which this linear system model is defined is determined either by completely specifying the state and control variables or by specifying an analysis point on a trajectory, selecting a trim option, and directing the program to determine the control variables and remaining state variables.

The system model determined by LINEAR consists of matrices for both the state and observation equations. The program has been designed to provide an easy selection and definition of the state, control, and observation variables to be used in a particular model. Thus, the order of the system model is completely under the user's control. Further, the program provides the flexibility of allowing alternate formulations of both the state and observation equations.

LINEAR has several features that make it unique among the standard linearization programs in the aerospace industry. The most significant of these features is flexibility. By generalizing the surface definitions and making no assumptions of symmetric mass distributions, the program can be applied to any aircraft in any phase of flight, except hover. The unique trimming capabability, provided by means of a user supplied subroutine, allow unlimited possibilities of trimming strategies and surface scheduling, which are particularly important for oblique-winged vehicles and aircraft having multiple surfaces effecting a single axis. The formulation of the equations of motion permit the inclusion of thrust vectoring effects. The ability to select, without program modification, the state, control, and observation variables for the linear models, which when combined with the large number of observation quantities available, allows any analysis problem to be attacked with ease.

This paper provides an introduction to the program LINEAR. The trimming capabilities of LINEAR are discussed from both a theoretical and implementation perspective. Time history comparisons of linear and nonlinear models of the high-performance aircraft were used as part of the validation of LINEAR and are presented to illustrate the capabilities of the program.

Program Overview

The program LINEAR numerically determines a linear systems model using nonlinear equations of motion and a user-supplied nonlinear aerodynamic model (Fig. 1). LINEAR is also capable of extracting linearized gross-engine effects, (such as net thrust, torque, and gyroscopic effects) and including these effects in the linear system model. The point at which this linear system model is defined is determined either by specifying the state and control variables or by selecting an analysis point on a trajectory, selecting a trim option, and allowing the program to determine the control variables and remaining state variables to satisfy the trim option selected.

Because the program is designed to statisfy the needs of a broad class of users, a wide variety of options has been provided. Perhaps the most important of these options are those that allow user specification of the state, control, and observation variables to be included in the model derived by LINEAR.

Within the program, the nonlinear equations of motion include 12 states representing a rigid aircraft flying in a stationary atmosphere over a flat, nonrotating earth. Internally, the state vector x is computed as

$$x = [p, q, r, V, \alpha, \beta, \phi, \theta, \psi, h, x, y]^T$$

The internal-control vector, u, can contain up to 30 controls. The internal observation vector, y, contains 120 variables including the state variables, the time derivatives of the state variables, the control variables, and a variety of other parameters of interest. Within the program,

$$y = \left[x^T, \dot{x}^T, u^T, y_1^T, y_2^T, y_3^T, y_4^T, y_5^T, y_6^T, y_7^T, y_8^T\right]^T$$

where

$$y_1 = \left[a_x, a_y, a_{n_x}, a_{n_y}, a_{n_z}, a_n, a_{n_{x,i}}, a_{n_{y,i}}, a_{n_{z,i}}, a_{n,i}, n\right]^T$$

$$y_2 = [a, Re, R\acute{e}, M, q, q_c, p_a, q_c/p_a, p_t, T, T_t, V_e, V_c]^T$$

$$y_3 = [\gamma, fpa, \dot{\gamma}, \ddot{h}, \dot{h}/57.3]^T$$

$$y_4 = [E_s, P_s]^T$$

$$y_5 = [L, D, N, A]^T$$

$$y_6 = [u, v, w, \dot{u}, \dot{v}, \dot{w}]^T$$

$$y_7 = [\alpha_{,i}, \beta_{,i}, h_{,i}, \dot{h}_{,i}]^T$$

and

$$y_8 = [T, p_s, q_s, r_s]^T$$

From the internal formulation of the state, control, and observation variables, the user must select the specific variables desired in the output linear model. Figure 2 illustrates the selection of the variables in the state vector for a requested linear model. From the internal formulation on the right, the requested model is constructed, and the linear system matrices are selected in accordance with the user specification of the state, control, and observation variables.

The model derived by LINEAR is determined at an analysis point. LINEAR allows this analysis point to be defined as a true, steady-state condition on a specified trajectory (a point at which the rotational and translational accelerations are zero) or a totally arbitrary state on an arbitrary trajectory. The program LINEAR provides the user with several options described in detail in the "Analysis-Point Definition" section of this paper. These analysis-point-definition options allow the user to trim the aircraft in "wings-level" flight, pushovers, pullups, level turns, or zero-sideslip maneuvers; also included is a nontrimming option in which the user defines a totally arbitrary condition about which the linear model is to be derived.

The linear system matrices are determined by numerical perturbation and are the first-order terms of a Taylor series expansion about the analysis point as described in the "Linear Models" section of this paper. The formulation of the output-system model is under user control. The user can select state-equation matrices corresponding to either the standard formulation of the state equation,

$$\dot{x} = Ax + Bu$$

or the generalized equation,

$$C\dot{x} = A'x + B'u$$

The observation matrices can be selected from either of two formulations corresponding to the standard equation,

$$y = Hx + Fu$$

or the generalized equation,

$$y = H'x + G\dot{x} + F'u$$

In addition to the linear system matrices, LINEAR also computes the nondimensional stability and control derivatives at the analysis point.

The input file for the batch version of LINEAR is a card-image file that defines the geometry and mass properties of the aircraft and selects various program options. Within this input file the state, control, and observation vectors desired in the output linear model are defined and the analysis-point options are selected. The interactive version of LINEAR allows user inputs from a terminal, the use of a card-image input file, or a combination of both.

The output of LINEAR is two files: one containing the linear system matrices and one documenting the options and analysis points selected by the user. The former of these files is intended to be used with follow-on design and analysis programs. The latter of these files contains all the information provided on the former file and also includes the details of the analysis point, the nondimensional stability, and control derivatives.

To execute LINEAR, four user-supplied subroutines are required. These subroutines define the nonlinear aerodynamic model, the gross-engine model, and the gearing between the LINEAR trim inputs and the surfaces modeled in the aerodynamic model. The gearing model, illustrated in Fig. 3, defines how the LINEAR trim inputs will be connected to the surface models and allows schedules and nonstandard trimming schemes to be employed.

Linear Models

The linearized system matrices computed by LINEAR are the first-order terms of a Taylor series expansion about the analysis point[2,3] and are assumed to result in a time-invariant linear system. The validity of this assumption is discussed in the "Analysis-Point Definition" section of this paper. The technique employed to obtain these matrices numerically is a simple approximation to the partial derivative,

$$\frac{\partial f}{\partial x} \approx \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x}$$

where f is a general function of x, an arbitrary, independent variable.

From the generalized nonlinear state,

$$T\dot{x} = f\ (x, \dot{x}, u)$$

and observation equations,

$$y = g\ (x, \dot{x}, u)$$

the program determines the linearized matrices for the generalized formulation of the system:

$$C\delta\dot{x} = A'\ \delta x + B'\ \delta u$$

$$\delta_y = H'\ \delta x + G\ \delta\dot{x} + F'\ \delta u$$

where

$$C = T - \frac{\partial f}{\partial \dot{x}}$$

$$A' = \frac{\partial f}{\partial x}$$

$$B' = \frac{\partial f}{\partial u}$$

$$H' = \frac{\partial g}{\partial x}$$

$$G = \frac{\partial g}{\partial \dot{x}}$$

and

$$F' = \frac{\partial g}{\partial u}$$

with all derivatives evaluated along the nominal trajectory defined by the analysis point $(x_0, \dot{x}_0, u_0)$ and the state, time derivative of state, and control vectors can be expressed as small perturbations about the nominal trajectory, so that,

$$x = x_0 + \delta x$$

$$\dot{x} = \dot{x}_0 + \delta\dot{x}$$

and

$$u = u_0 + \delta u$$

In addition to the matrices for the generalized system just described, the user has the option of requesting linearized matrices for a standard formulation of the system:

$$\delta\dot{x} = A\ \delta x + B\ \delta u$$

$$\delta y = H\ \delta x + F\ \delta u$$

where

$$A = \left[T - \frac{\partial f}{\partial \dot{x}}\right]^{-1} \frac{\partial f}{\partial x}$$

$$B = \left[T - \frac{\partial f}{\partial \dot{x}}\right]^{-1} \frac{\partial f}{\partial u}$$

$$H = \frac{\partial g}{\partial x} + \frac{\partial g}{\partial \dot{x}} \left[T - \frac{\partial f}{\partial \dot{x}}\right]^{-1} \frac{\partial f}{\partial x}$$

and

$$F = \frac{\partial g}{\partial u} + \frac{\partial g}{\partial \dot{x}} \left[T - \frac{\partial f}{\partial \dot{x}}\right]^{-1} \frac{\partial f}{\partial u}$$

with all derivatives evaluated along the nominal trajectory defined by the analysis point $(x_0, \dot{x}_0, u_0)$.

By determining the matrices of the generalized formulation of the system, LINEAR avoids the problems inherent in systems having the nonlinear formulation

$$\dot{x} = f(x, \dot{x}, u)$$

$$y = g(x, \dot{x}, u)$$

rather than

$$\dot{x} = f(x, u)$$

$$y = g(x, u)$$

The latter formulation is often assummed implicitly, although for most aircraft the former formulation is correct particularly because of the presence of the aerodynamic derivatives with respect to angle-of-attack rate ($\dot{\alpha}$) and angle of sideslip rate ($\dot{\beta}$). If the system matrices of the standard formulation of the linear system, given by

$$\dot{x} = Ax + Bu$$

$$y = Hx + Fu$$

are determined directly from nonlinear functions having state rate ($\dot{x}$) dependence, an iterative scheme must be employed with each perturbation of the state and control variables while computing the partial derivatives. This is not always done, making the resultant linear system matrices incorrect.

LINEAR also provides two nonstandard matrices, D and E, in the equations,

$$\dot{x} = Ax + Bu + D\ v$$

$$y = Hx + Fu + E\ v$$

or, D′ and E′, in the equations,

$$C\dot{x} = A'x + B'u + D'v$$

$$y = H'x + G\dot{x} + F'u + E'v$$

These dynamic interaction matrices include the effect of external forces and moments acting on the vehicle. The components of the dynamic interaction vector, v, are incremental body-axis forces and moments:

$$v = \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \\ \delta L \\ \delta M \\ \delta N \end{bmatrix}$$

Thus

$$D' = \frac{\partial f}{\partial v}$$

and

$$E' = \frac{\partial g}{\partial v}$$

$$D = \left[T - \frac{\partial f}{\partial \dot{x}}\right]^{-1} \frac{\partial f}{\partial v}$$

and

$$E = \frac{\partial g}{\partial v} + \frac{\partial g}{\partial \dot{x}} \left[T - \frac{\partial f}{\partial \dot{x}}\right]^{-1} \frac{\partial f}{\partial v}$$

The purpose of these matrices is to allow the effects of unusual subsystems or control effectors to be easily included in the vehicle dynamics.

The default output matrices for LINEAR are those for the standard system formulation. However, the user can select matrices for either the generalized or the standard state and observation equations in any combination. Internally, the matrices are computed for the generalized system formulation and then combined appropriately to accommodate the system formulation requested by the user.

Analysis-Point Definition

The point at which the nonlinear system equations are linearized is referred to as the analysis point. This point can represent a true, steady-state condition on the specified trajectory (a point at which the rotational and translational accelerations are zero)[4,5] or a totally arbitrary state on a trajectory. LINEAR allows the user to select from a variety of analysis points. Within the program, these analysis points are referred to as trim conditions, and several trim options are available to the user. The arbitrary state and control option is designated NOTRIM, and in selecting this option the user must specify all nonzero state and control variables. For the equilibrium conditions, the user specifies a minimum number of parameters and the program numerically determines required state and control variables to force the rotational and translational accelerations to zero. The analysis-point options are described in detail in the following subsections.

For all of the analysis-point definition options, any state or control parameter may be input by the user. Those state variables not required to define the analysis point are used as initial estimates for the calculation of the state and control conditions that result in zero rotational and translational accelerations.

It should be noted that the option of allowing the user to linearize the system equations about a nonequilibrium condition raises theoretical issues beyond the scope of this report, but of which the potential user should be aware. While all of the analysis-point definition options provided in LINEAR have been found to be useful in the analysis of vehicle dynamics, not all of the linear models derived about these analysis points result in the time-invariant systems assumed in this report. However, the results of the linearization provided by LINEAR do give the appearance of being time invariant.

The linearization process as defined in this report is always valid for some time interval beyond the point in the trajectory about which the linearization is done. However, for the resultant system to be truly time invariant the vehicle must be in a sustainable, steady-state flight condition. This requirement is something more than merely a trim requirement, which is typically represented as

$$\dot{x}(t) = 0$$

indicating that for trim, all the time derivatives of the state achieved must be zero. (This is not the case, however: Trim is achieved when the acceleration-like terms are identically zero; no constraints need to be placed on the velocity-like terms in $\dot{x}$. Thus, for the model used in LINEAR, only $\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{V}$, $\dot{\alpha}$, and $\dot{\beta}$ must be zero to satisfy the trim condition.) The trim condition is achieved for the straight and level, pushover/pullup, level turn, thrust-stabilized turn, and the beta-trim options listed below. In general, the analysis-point-definition options for NOTRIM and specific-power conditions do not result in a trim condition.

Of these analysis-point options resulting in a trim condition, only the straight-and-level and level-turn options force the model to represent sustainable flight conditions. In fact, only in the special case where the flightpath angle is zero does this occur for these options.

As stated earlier, the linearization of a nonlinear model and its representation as a time-invariant system is always valid for some time interval beyond the analysis point on the trajectory. This time interval is determined by several factors such as trim, sustainable flight conditions, and in the end by accuracy requirements placed on the representation. Thus, in using the linear models provided by this program, the user should exercise some caution.

Untrimmed

For the untrimmed option, the user specifies all state and control variables that are to be set at some value other than zero. The number of state variables specified is entirely at the user's discretion. If any of the control variables are to be nonzero, the user must specify the control parameter and its value. The untrimmed option allows the user to analyze the vehicle dynamics at any flight condition, including transitory conditions.

Straight-and-Level Trim

The straight-and-level trims available in LINEAR are constant flightpath-angle trims at "wings-level." Both options available for straight-and-level trim allow the user to specify either a flightpath angle or an altitude rate. However, since the default value for these terms is zero, the default for both types of straight-and-level trim is wings-level, horizontal flight.

The two options available for straight-and-level trim require the user to specify altitude and either an angle of attack or Mach number. If a specific angle-of-attack and altitude combination is desired LINEAR determines the velocity required for the requested trajectory. LINEAR also takes the user's specified Mach number and altitude, and uses the trim routines in the program to determine the angle of attack needed for the requested trajectory.

Pushover/Pullup

The analysis-point-definition options for pushover/pullup result in wings-level flight at load factors of greater than or less than one. For the elevated-load-factor case, the analysis point is the minimum altitude point of a pullup when altitude rate is zero. For the case of load factor less than one, this point results in a pushover at the maximum altitude with an altitude rate of zero. There are two options available for the analysis-point-definition of pushover/pullup: (1) in which angle of attack is determined from the specified altitude and Mach number, and (2) in which angle of attack, altitude, and Mach number are specified and load factor is determined according to the constraint equations.

Level Turn

The analysis-point-definition options for level turn result in "nonwings-level," constant-turn-rate flight at load factors greater than one. The vehicle model is assumed to have sufficient excess thrust to trim at the condition specified. If thrust is not sufficient, trim will not result and the analysis point thus defined will have a nonzero (in fact, negative) velocity rate.

The level trim options available in LINEAR require the specification of an altitude and Mach number. The user can then use either angle of attack or load factor to define the desired flight condition. For either of these options the user may also request a specific flightpath angle or altitude rate. Thus, these analysis-point definitions may result in ascending or descending spirals, although the default is for the constant altitude turn.

Thrust-Stabilized Turn

The analysis-point-definition for a thrust-stabilized turn results in a constant-throttle "nonwings-level" turn with a nonzero altitude rate. The two options available allow the user to specify either the angle of attack or the load factor for the analysis point. The altitude and Mach number at the analysis point must be specified for either of the two options. The user also must specify the value of the thrust-trim parameter.

Beta Trim

The analysis-point-definition option for beta trim results in "nonwings-level," horizontal flight with a zero-heading rate at a user-specified Mach number, altitude, and angle of sideslip. This trim option is nominally at 1 g, but as beta varies from zero, normal acceleration decreases and lateral acceleration increases. For an aerodynamically symmetric aircraft, a trim-to-zero beta using the beta-trim option results in the same trimmed condition as the straight-and-level trim. However, for an asymmetric aircraft such as an oblique-winged vehicle, the two trim options are not equivalent.

Specific Power

The analysis-point-definition option for specific power results in a level turn at a user-specified Mach number, altitude, thrust-trim parameter, and specific power. Unlike the other trim options provided in LINEAR, the specific power option in general does not attempt to achieve a zero velocity rate. Because the altitude rate is zero and specific power is defined by

$$P_s = \dot{h} + \frac{V \dot{V}}{g}$$

the resultant velocity rate, $\dot{V}$, will be

$$\dot{V} = \frac{P_s g}{V}$$

However, the other acceleration-like terms ($\dot{p}$, $\dot{q}$, $\dot{r}$, $\dot{\alpha}$, and $\dot{\beta}$) will be zero if the requested analysis point is achieved.

Program Validation

Two methods were used to validate LINEAR. The first of these methods required that the elements of the matrices of the generalized, linear system equations be derived analytically. These results were compared to the numerically derived matrix elements produced by LINEAR. The second method of validation was by comparison of linear and nonlinear time histories. The linear time histories were generated by solving the linear system differential equations using

$$x(t) = x_0 + e^{At}\,\delta x_0 + \int_0^t e^{A(t-\tau)}B\,\delta u(\tau)\,d\tau$$

and

$$y(t) = y_0 + He^{At}\,\delta x_0 + \int_0^t He^{A(t-\tau)}B\,\delta u(\tau)\,d\tau + F\,\delta u(t)$$

The nonlinear time histories were generated using independent implementations of the equations of motion and sensor models. However, the nonlinear simulations and the linearized matrices were based on the same implementation of the nonlinear aerodynamic models. The nonlinear simulations used a 12-state model. The linear simulations used a nine-state model excluding Ψ, x, and y.

The simulation used for the time history comparisons was based on a model of a two-engine, high-performance aircraft capable of speeds in excess of 2.0 Mach number and altitudes in excess of 50,000 ft. Two sets of time histories are presented in Figs. 4 and 5. The first time history is based on a straight-and-level trim at 0.7 Mach number at 20,000-ft altitude. Figure 4 includes a time history from a four-state linear model in addition to the nonlinear and nine-state linear models. For the time history shown in Fig. 4 this model was

$$x = \begin{bmatrix} q \\ V \\ \alpha \\ \theta \end{bmatrix}$$

Figure 4 shows the results of a ±2° elevator doublet. The first feature to note about the time histories in Fig. 4 is that except for the velocity and altitude time histories, there is very little difference between the response of the three models. The second feature to note about these time histories is that, except for the altitude response, the performance of the nine-state and four-state models give identical results. Figure 5 is based on a 3-g level turn at 0.9 Mach number at 35,000-ft altitude with a ±2° aileron doublet. Here, the time histories for the lateral-directional parameter are almost identical, except for the bank angle; the longitudinal-parameter time histories, while in reasonable agreement, show steady-state divergence, but with similar transient response.

The comparison of linear and nonlinear models in Figs. 4 and 5 illustrate two of the trim options available in LINEAR. These time histories also demonstrate the ability of LINEAR to derive linear aircraft models. The lack of complete agreement between the linear and nonlinear reflects on the adequacy of using linear models rather than on LINEAR. If linear models are desired, the program LINEAR provides a useful tool for generating them.

Concluding Remarks

The FORTRAN program, LINEAR, was developed to provide a flexible, powerful, and documented tool to derive linear models for aircraft-stability analysis and control law design. The program LINEAR numerically determines a linear systems model using nonlinear equations of motion and a user-supplied, nonlinear, aerodynamic model. LINEAR is also capable of extracting both linearized engine effects (such as net thrust, torque, and gyroscopic effects) and including these effects in the linear system model. The point at which this linear system model is defined is determined either by completely specifying the state and control variables or by specifying an analysis point on a trajectory, selecting a trim option, and directing the program to determine the control variables and remaining state variables.

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LINEAR has several features that make it unique among the linearization programs common in the aerospace industry. The most significant of these features is flexibility. By generalizing the surface definitions and making no assumptions of symmetric mass distributions, the program can be applied to any aircraft in any phase of flight, except hover. The unique trimming capabability, provided by means of a user-supplied subroutine, allow unlimited possibilities of trimming strategies and surface scheduling which are particularly important for oblique-winged vehicles and aircraft having multiple surfaces affecting a single axis. The formulation of the equations of motion permit the inclusion of thrust-vectoring effects. The ability to select, without program modification, the state, control, and observation variables for the linear models, which when combined with the large number of observation quantities available, allows any analysis problem to be attacked with ease.

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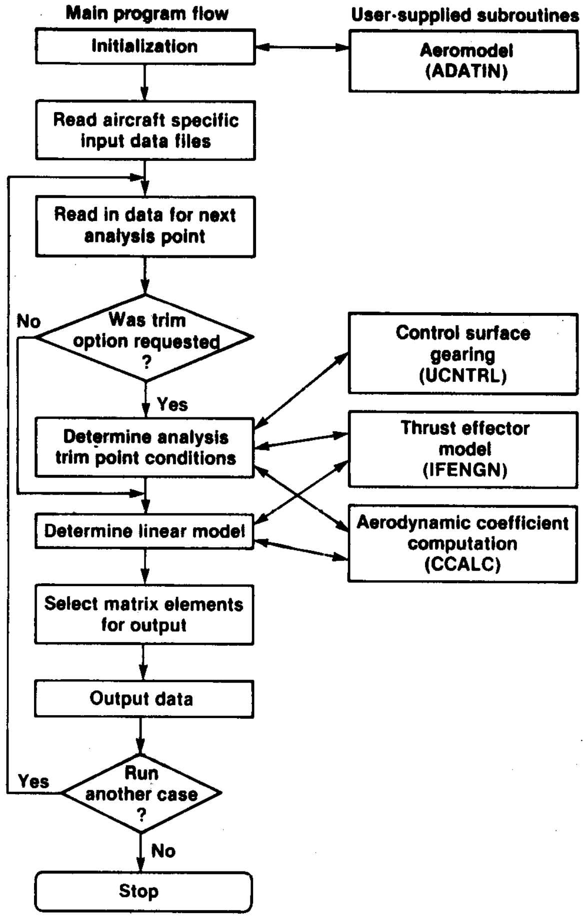

Fig. 1 Program flow diagram showing communication with user-supplied subroutines.

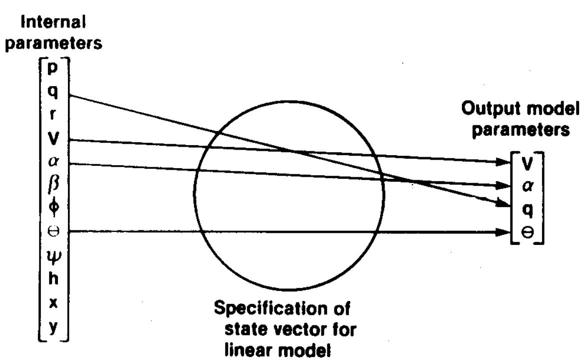

Fig. 2 Selection of state variables for linear model.

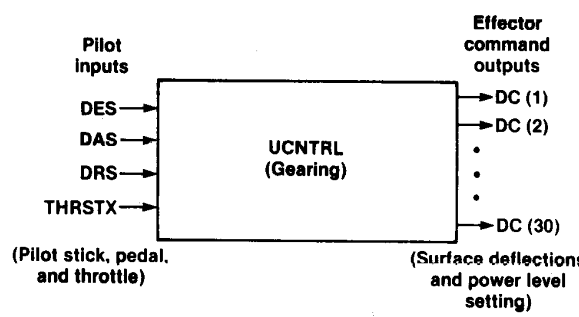

Fig. 3 Inputs and outputs to the user-supplied subroutine UCNTRL.

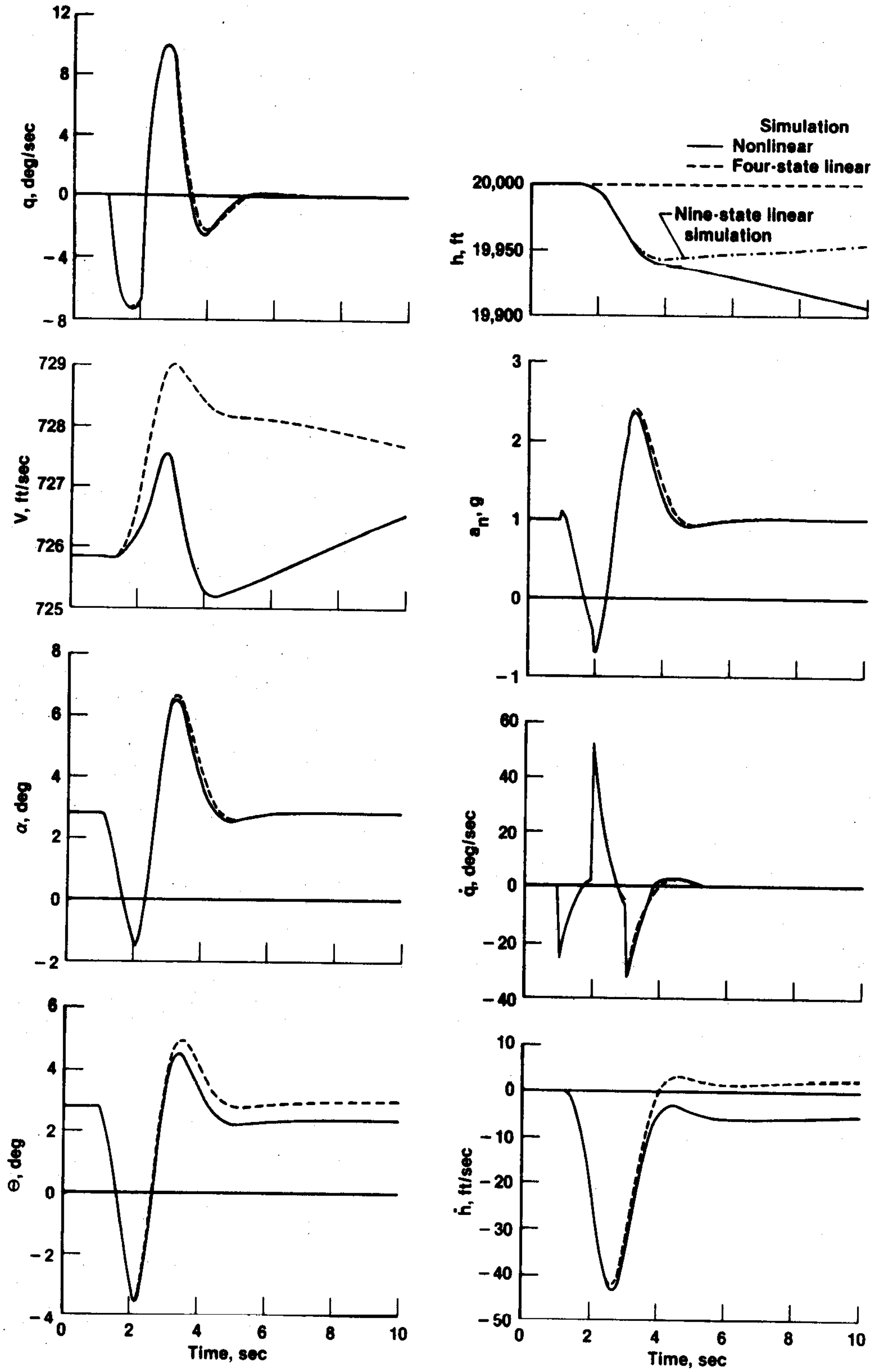

Fig. 4 Comparison of nonlinear and linear simulations of a high-performance aircraft for an elevator doublet; level flight; 0.7 Mach number; at 20,000 ft altitude.

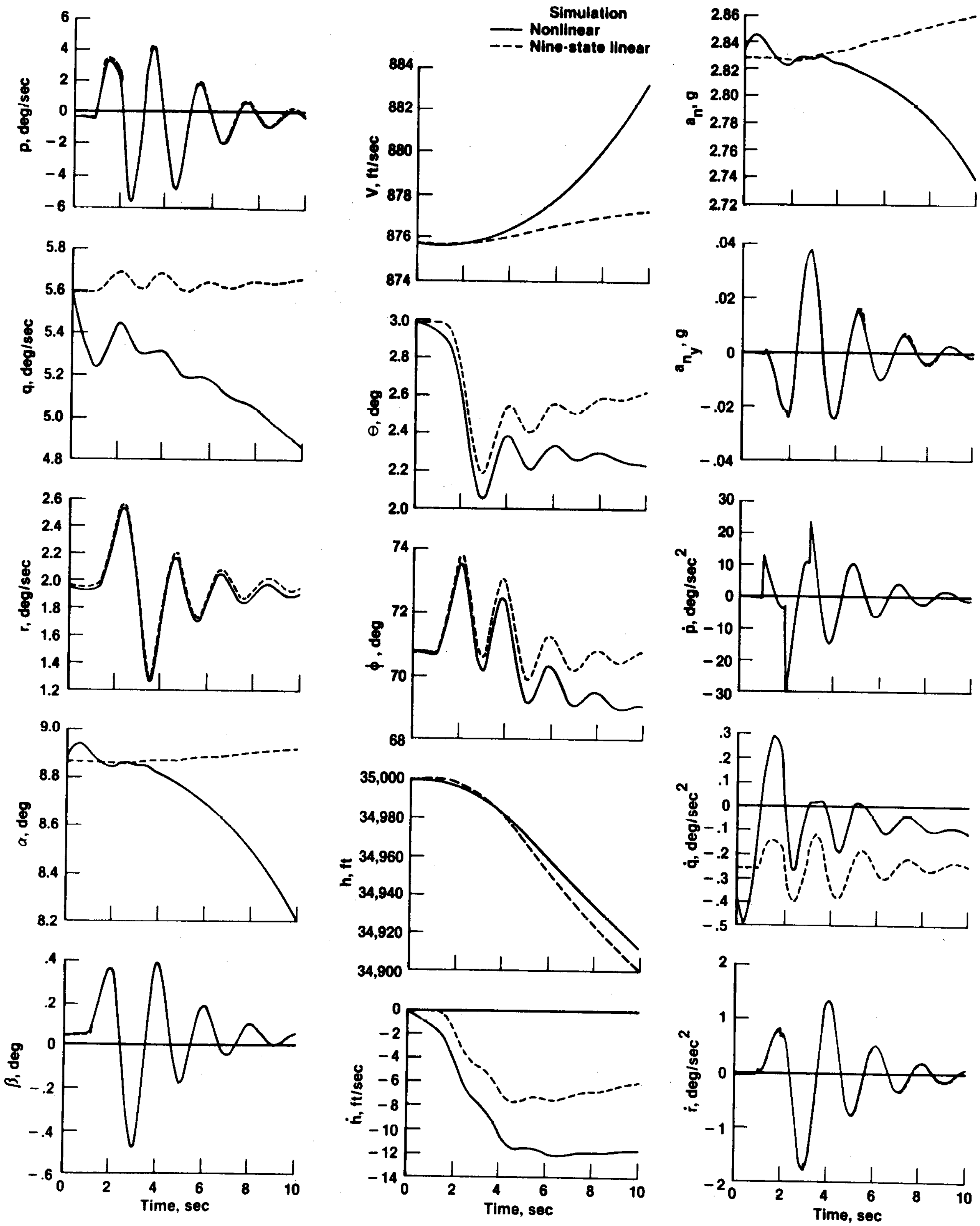

Fig. 5 Comparison of nonlinear and linear simulations of a high-performance aircraft for an aileron doublet; 3g level turn; 0.9 Mach number; at 35,000 ft altitude.

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16. Abstract

This paper discusses a FORTRAN program that provides the user with a powerful and flexible tool for the linearization of aircraft models. The program LINEAR numerically determines a linear systems model using nonlinear equations of motion and a user-supplied, nonlinear aerodynamic model. The system model determined by LINEAR consists of matrices for both the state and observation equations. The program has been designed to allow easy selection and definition of the state, control, and observation variables to be used in a particular model. Also, included in the report is a comparison of linear and nonlinear models for a high-performance aircraft.

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